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A Method for Modal Correction between Simulation and Test

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Abstract: This paper introduces a common modal correction method, Modal Assurance Criterion. But it has a problem that it cannot handle the issue of the reversed mode. In this present work, when the rudder rotates, it vibrates diversely. The modal shapes of case One are bending first, and twist second. The other ones are twist first, and bending second. And therefore, the modal shapes are applied to optimize, according to the different modal orders. The proposed method of sequentially comparing every modal shape could find potential applications in dynamics problems.
Keywords: Mode correction; Modal assurance criterion (MAC); Modal testing; Error

Nomenclature

$\{\varphi_x\}_q$ test modal vector, mode q

$\{\varphi_a\}_r$ compatible analytical modal vector, mode r

$\{\varphi_x\}_q^T$ transpose of $\{\varphi_x\}_q$

$\{\varphi_a\}_r^T$ transpose of $\{\varphi_a\}_r$

1. Introduction

Modal analysis is a method for studying structural dynamic characteristics and is generally used in the field of engineering vibration. Among them, the mode refers to the natural vibration characteristics of the mechanical structure, and each mode has a specific natural frequency, a damping ratio, and a modal shape. The process of analyzing these modal parameters is called modal analysis. According to the calculational method, modal analysis can be divided into computational modal analysis and experimental modal analysis.

The calculated modal analysis is gotten by Finite Element Method; each order corresponds to one mode, which has its own specific frequency, damping, and modal parameters. Through the test, the input and output signals of the collected system are obtained through parameter identification - experimental modal analysis.

The vibration mode is an inherent, integral property of the elastic structure. Through the modal analysis method, the characteristics of the main modes of the structure in a certain vulnerable frequency range are clarified, and it is possible to predict the actual vibration response of the structure under the influence of external or internal vibration sources in this frequency band. Therefore, modal analysis is an important method for structural dynamic design and equipment fault diagnosis.

The actual vibration modes of machines, buildings, aerospace vehicles, ships, automobiles, and etc. vary. Modal analysis provides an effective way to study various types of vibration characteristics. The ultimate goal of modal analysis is to identify the modal parameters of the system, and provide a basis for structural vibration analysis, vibration fault diagnosis and prediction, and optimal design of structural dynamic characteristics.

The application of modal analysis techniques can be attributed to the following aspects:

1) Evaluate the dynamic characteristics of existing structural systems;
2) Predicting and optimizing the structural dynamic characteristics in new product designs;
3) Diagnose and predict structural system failures;
4) Control structure radiation noise;
5) Identify the load of the structural system.

Modal parameters include modal frequencies, modal masses, modal vectors, modal stiffness, and modal damping. Ideally we would like to get a complete set of modes for the structure, which is impossible and unnecessary in practical applications. In fact not all modes contribute the same to the response. For the low-frequency response, the amplitude is the largest which is the most dangerous, and the high-frequency amplitude is small, so the influence of the higher-order modes is small. For the actual structure, we are often interested in its front tens of modes, and higher modes are often abandoned. Although this will cause a little error, the matrix order of the frequency response function will be greatly reduced, and the workload will be greatly reduced. This is called modal truncation. Therefore, many dynamic problems consider the low-order vibration modes.

In general, experimental modal analysis and finite element modal analysis are used in combination.

1) Use the finite element analysis model to determine the measurement points, excitation points, and support points (suspension points) of the modal test, and identify and name the vibration mode of test modal parameters, especially for complex structures.
2) Use the test results to modify the finite element analysis model to meet industries or national standards requirements.
3) The finite element model is used to simulate and analyze the error caused by the test conditions, such as the error caused by boundary condition simulation, additional mass, and additional stiffness, and its elimination.
4) Using finite element model analysis to solve problems in the experiment.
5) Analysis of spectral consistency and mode vector correlation of the two sets of models. Generally, we use Modal Assurance Criterion (MAC) to compare the mode vectors of them. And furthermore, it has been the most efficient and most widely used tool for modal corrections up to now. In this paper, advantages and disadvantages of it are discussed. And a new approach is proposed.

MAC has been proposed since the late 1970s, which was used in the area of experimental and analytical structural dynamics. The MAC was originally introduced in modal testing in connection with The Modal Scale Factor, as an additional confidence factor in the evaluation of modal vector from different excitation locations. The MAC is calculated as follows

$$\mathrm{MAC(r,q)} = \frac{\left|\{\varphi_a\}_r^T\{\varphi_x\}_q\right|^2}{(\{\varphi_a\}_r^T\{\varphi_a\}_r)(\{\varphi_x\}_q^T\{\varphi_x\}_q)} \tag{1}$$

If the absolute value of the sum of product elements is squared, the above formulation is written by

$$\mathrm{MAC(a,x)} = \frac{\left|\sum_{j=1}^{n}\{\varphi_a\}_j\{\varphi_x\}_j\right|}{\left(\sum_{j=1}^{n}\{\varphi_a\}_j^2\right)\left(\sum_{j=1}^{n}\{\varphi_x\}_j^2\right)} \quad (2)$$

Where n is the number of the counted modes. For complex modes of vibration, $\{\varphi_x\}_q$ and $\{\varphi_a\}_r$ are complex numbers.

After decades of use, misuse and abuse appear more and more prominent. Professor Randall J. Allemang[1] pointed out that it would continue to be developed, though it had its own strengths and weaknesses. He thought the usual befits lie in

1) Validation of experimental modal models
2) Correlation with analytical modal models (mode pairing)
3) Correlation with operating response vectors
4) Mapping matrix between analytical and experimental modal models
5) Modal vector error analysis
6) Modal vector averaging
7) Experimental modal vector completion and/or expansion
8) Weighting for model updating algorithms
9) Modal vector consistency/stability in modal parameter estimation algorithms
10) Repeated and pseudo-repeated root detection
11) Structural fault/damage detection
12) Quality control evaluations
13) Optimal sensor placement

However, several issues had been summarized by him as

1) The modal analysis criterion is not an orthogonality check.
2) The wrong mathematical formulation for the modal assurance criterion is used.
3) The modal assurance criterion is sensitive to large values (wild points?)and insensitive to small values.
4) The number of elements in the modal vectors (space) is small.
5) The modal vectors have been zero padded.

In the historical development of MAC, a lot of improved methods came up one after another, such as the coordinate modal assurance criterion (COMAC), the frequency response assurance criterion (FRAC), coordinate orthogonality check (CORTHOG), frequency scaled modal assurance criterion (FMAC), partial modal assurance criterion (PMAC), scaled modal assurance criterion (SMAC), and modal assurance criterion using reciprocal modal vectors (MACRV). These methods all had been largely enhanced, which also perfected the judgment ability of them for modal problems.

As can be known, the MAC has been widely used to predict the structural dynamic characteristics of various systems since it was proposed, for example, correcting a mass or stiffness matrix. In 2012, Miroslav Pastor, Michal Binda, and Tomáš Hacrarik [2] reviewed the using of the original MAC. They thought it was a statistical indicator that was most sensitive to large differences and relatively insensitive to small differences in the mode shapes. The MAC was often used to pair modes shapes derived from analytical models with those obtained experimentally. It was easy to apply and did not require an estimate of the system matrices. It was bounded between 0 and 1, with 1 indicating fully consistent mode shapes. It could only indicate consistency and did not indicate validity or orthogonality. A value near 0 indicated that the modes were not consistent.

In 2013, Masayoshi Misawa and etc. [3], showed the other method for modal correction and proposed the new conception of the weighting coefficient which was involved in both mass additive locations and the prediction of dynamic characteristics of structures. If it was selected appropriately, the accurate frequency and mode of structures would be obtained through component model tests.

In 2014, Modak S. V. [4], distinguished the good matrices after updating from the bad ones before by use of MAC values, which was still employed to determine the correlation of the structural modes.

In 2015, Sairajan K. K. [5], proposed base force assurance criterion indicating that the force is transmitted to the base during the base excitation test. The criterion can predict the error in the natural frequencies of the fundamental modes. And it also saved the model testing cost.

In 2016, Mercer J. F. [6], further studied the influence of sensor placement on reduced model quality. MAC assessments were performed to determine which reduced mode matched most closely the corresponding selected full target mode. The MAC checks showed significantly lower off diagonals with the selected DOFs than with those eliminated in the effective independence selection.

Over the years, the MAC method has been applied to various dynamic issues, such as frequency response, flutter analysis, sensor placement, and so on. Since the MAC for modal corrections cannot deal with the trouble of the reversed modes of the rudder for flutter in this paper, using modal shape optimization method to improve the prediction precision of the flutter analysis is illustrated. And it seems to be extended to more model modifications for dynamic problems in the close future.

In the present research, the stiffness of the rudder axis could be adjusted during the flight or the experiment, which changes the modes of it. Most of these methodologies require either an accurate damping and frequency estimation of the physical modes or accurate aeroelastic model estimation. In other words, the accuracy of the flutter-prediction results of these methods heavily depends on how good the model or mode estimations are. Therefore, the modes of the rudder directly influencing flutter speeds become very critical. As usual, the MAC method is used to compare some simulated modal shapes with the data from the ground vibration test (GVT), but in those two studied cases, it cannot distinguish the order of vibration modes.

So, firstly, unlike traditionally aeroelastic systems of maximum performance and minimum weight, studied by Melike Nikbay [7], and Muhammet N. Kuru, 2013, the objective was a minimal 1-order or 2-order error which was a ratio of test eigenvectors and calculated ones, constraints were the first and second natural frequencies, and design variables were stiffness of the different rudder axles. Finally, modes of the first bending and second twisting of the rudder were found out in Case one, and these of the first torsion and second bending were extracted in Case two, though the first and second frequencies were similar in them.

In addition, the current study focuses on finding a right analysis model by an optimization method, which is proposed in this paper. Test investigations are first presented, followed by verifying the structural model and mode by optimization. Although two flutter cases both have the same flutter mechanism, classical bending-torsion flutter, they have their own critical flutter points, respectively.

2. Experimental Data

As we know, flutter, where in a certain phasing of elastic, inertial, and aerodynamic loads causes a structure to oscillate in an unbounded manner (i.e., dynamical instability), is of particular concern from a safety vantage point. Aeroelastic flutter occurs at any unpredictable time. However, it is impossible to completely predict the exact flutter point, due to the limitation of various factors. From some perspective, flight flutter testing brings some possibility to verify the simulation. It, however, is an expensive and hazardous task, and required to verify if the aircraft is truly free from flutter within the margin of the aircraft's operational envelope.

In order to validate the supersonic flutter calculation method and the verification of the flutter model accuracy, done by Bingyuan Yang [8], Weili Song 2001, in FD-06 wind tunnel of 701 Institute, a test of a rudder was conducted. The data of eigenvector records taken in ground vibration test (GVT) were being preserved and would be available for further analysis. All test conditions and model configurations for available data are summarized in Table 1.

Table 1: Data of GVT

Case 1 Test Number: 2	Mach number=1.53 Angle of attack=2º10´ 1^{st} order frequency:35.86 2^{nd} order: 64.74
Case 2 Test Number:13	Mach number=2.51 Angle of attack=4º50´ 1^{st} order frequency:29.68 2^{nd} order: 58.86

In the table above, the control points of vibration shapes are shown in Fig. 1.

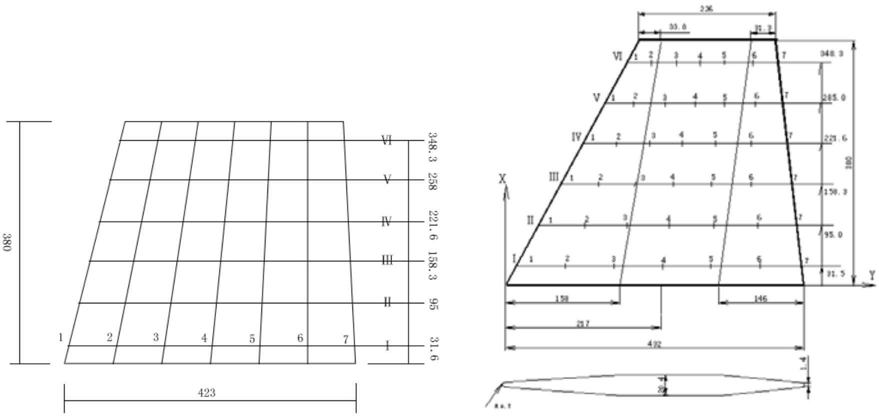

Figure 1: Location of monitoring points for eigenvectors (Unit: millimeter)

Table 2 shows the flutter results via this test.

Table 2: Test parameters

Case	Points*1	Mach	Angle of attack	Static Pressure*2 (Pa)	Temperature (K)	Density (Kg/m^3)	Speed of sound (m/s)
1	2	1.53	2º10´	33230	197.215	0.5873	281.44
2	13	2.51	4º50´	15457	127.720	0.40132	226.49

*1 Points refer to the number of the test.

*2 the initial angle causes static pressure.

3. Model/Mode Correction

The following section introduces the details of the rudder geometry model in Fig. 2 used in the test and the generation of an accurate FEM (Finite Element Method), which matches experimental data.

3.1 Rudder Model

The rudder axle in Fig. 2 was attached to the pitch and twist springs. The outer leaves of the springs were rigidly mounted to the tunnel walls. The pitch springs were commercially available flexure pivots that allowed ±7.5^0 of rotation through the elastic bending of rudder flexures. The twist springs had a flexural rigidity range. Usually, the spring retained its linear characteristics over large displacements. Also, there were no sliding surfaces at support points which would produce damping. The springs were designed to deflect elastically approximately 10 mm. During flight or in the wind tunnel, rudder stiffness uncertainties are related to spring rotation and operating conditions. In particular, some record data from the experiment are not so reliable, maybe, from immature operations or instrumental limitations. So, it is naturally involved in designing and optimizing a realistic structure for a required level of reliability and efficiency for supersonic flutter validation.

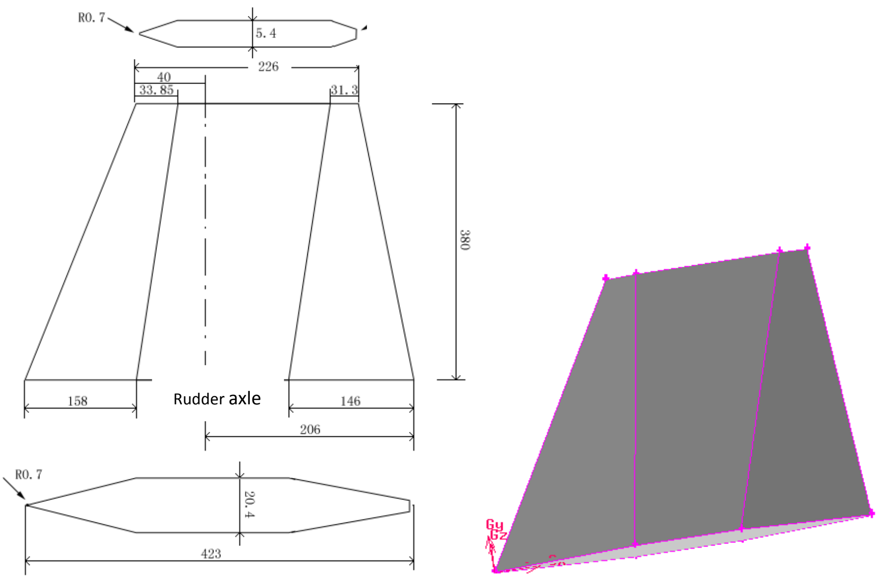

Figure 2: Rudder Model (Unit: millimeter)

3.2 FEM and Boundary Conditions

The rudder was composed of a Ti-alloy material with the rudder torque tube that was made of a steel material. The material parameters are listed in Table 3.

Table 3: Structural Material Parameters

Parameters	Young module （MPa）	Poisson ratio	Density （Kg/m^3）
Rudder	4.1e10	0.34	1810.
Axle	2.11e11	0.3	7800.

The finite element model (see Fig. 3) was composed of 540 hexahedral structural elements. The rudder torque tube was modeled with 10 linear beam elements.

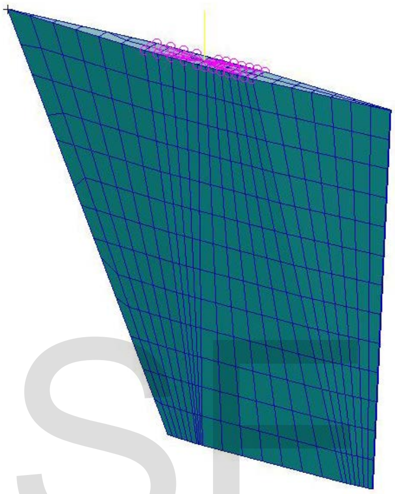

Figure 3: FEM of rudder

The node at the end of the rudder shaft was fixed in 3 translations and 2 rotations, but was free to rotate about the axis of the shaft.

3.3 Mode Correction by MAC

After the modal analysis of the initial rudder model by MSC.Nastran, the MAC was used to pair modes shapes derived from analytical models with those obtained experimentally. It indicated practically consistent mode shapes from Case One and Case Two. Obviously, these two cases should be different models for flutter analyses, due to the different flutter speeds of two tests. And therefore, the other method is supposed to deal with it.

3.4 Mode Correction by Optimization Method

In the present research, Sequential Quadratic Programming (SQP), also known as Quadratic Approximation, is applied, whose outstanding strongpoint is the less number of function and gradient evaluation, and the higher computational efficiency, especially for the rudder structural optimization objective function being a linear or nonlinear function of the design variables, and constraints such as frequencies for a function of them. Using the method to optimize its stiffness, applying the mode shape error derivative concept to calculate the sensitivity of the stiffness, and employing this Method in the iterative process make rapid the optimization convergence, and reliable the computational results.

3.4.1 Parameters of Optimization

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When the optimal model was established, it was possible to define the optimization conditions and perform mode analysis.

1) The objective function:

From Table 1, the vibration amplitude distribution of the first frequency of the rudder was nonlinear in Case 1, and therefore, the eigenvector error equation was written by

$$error1 = \sqrt{\sum_{i=1}^{n} (x_i - y_i)^2 / n} \quad (3)$$

Where g = Damping; n = Sample number; x_i = Calculated value of eigenvector corresponding to point i; y_i = Test value of eigenvector corresponding to point i. n sample nodes were located at the same location of the test points from the upper rudder surface in figure 4.

Figure 4: Monitoring nodes of FEM

For Case 2, the eigenvector error equation was defined by linear, due to the linear changing of the vibration displacements.

$$error2 = \sum_{i=1}^{n} |x_i - y_i| / n \quad (4)$$

The two errors in modal analysis of the rudder were the objectives to be minimized in this research, respectively.

2) Constraints:

The first and second frequencies were taken as constraints. At the same time, the stiffness of the rotation shaft was also restricted to some limits.

3) Design variables:

The stiffness of the rudder axle was defined as design variables.

3.4.2 Optimization Results

The optimization typically took 9 and 26 iterations, for Case 1 and Case 2 respectively to converge to the precision required for the gradient optimization. They are shown in Figs. 5 and 6.

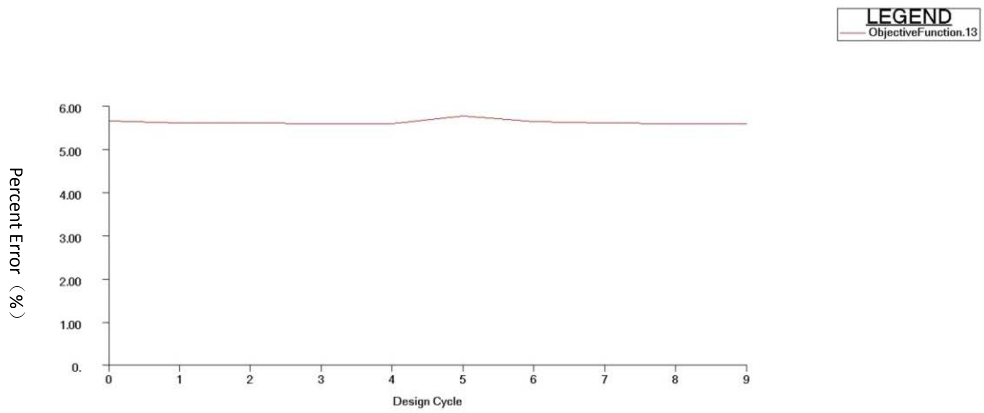


Fig. 5: History of Case 1（Vertical axis shows percent error in Eq. 3）

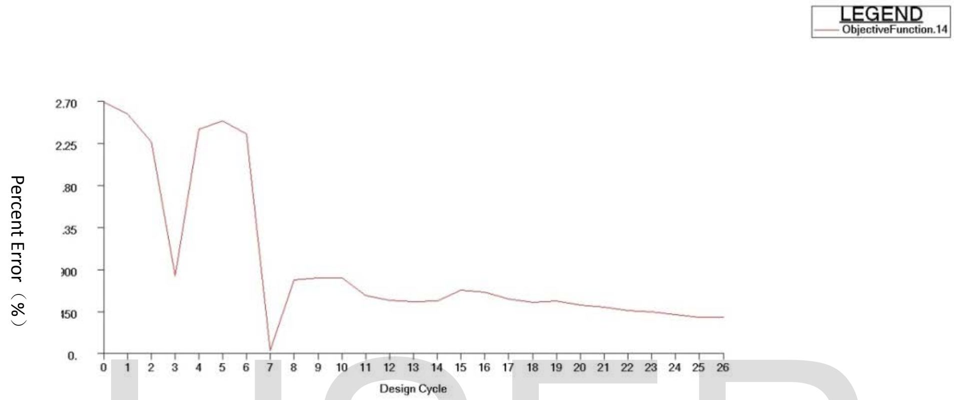


Fig. 6: History of Case 2 (Vertical axis shows percent error in Eq. 4)

In Case 1, the first mode surface was fitted in the test data and the optimization monitoring points in figures 7 and 8.

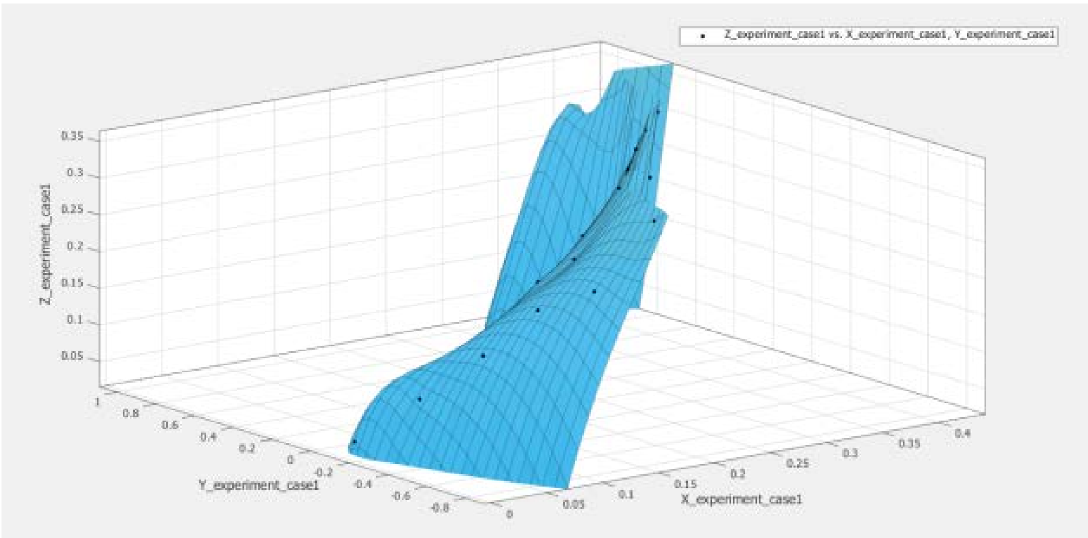


Figure 7: First mode surface of the test data

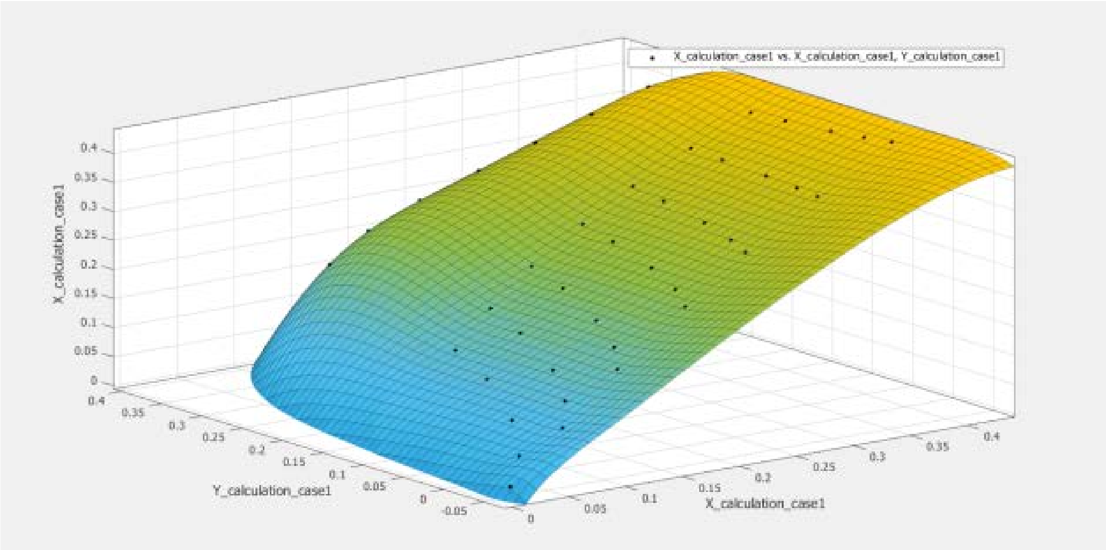


Figure 8: First mode surface of the optimization monitoring points

The figure 7 above shows that the experimental points change dramatically and disorderedly, while the optimized ones in figure 8 transit gradually and softly. It also demonstrates the 2nd-order error for the first mode shape in Case 1 is selected properly.

In Case 2, the first mode surface is fitted in the test data and the optimization monitoring points in figures 9 and 10.

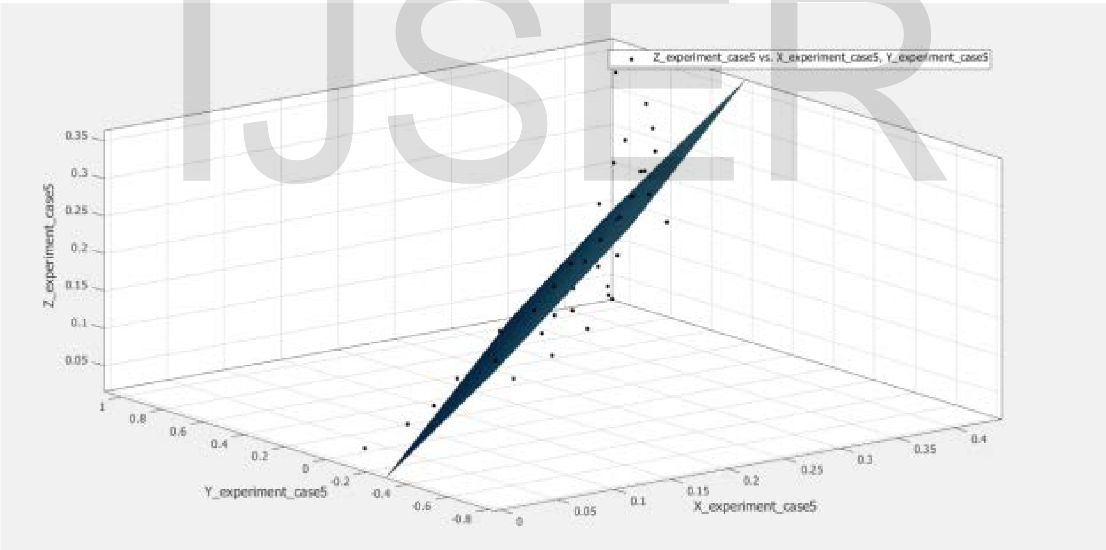


Figure 9: First mode surface of the test data

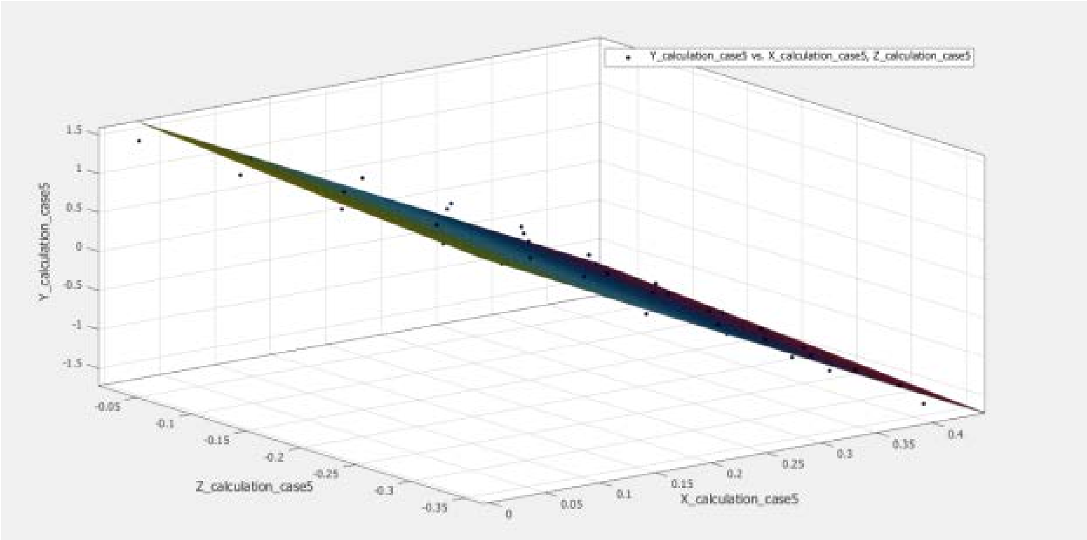


Figure 10: First mode surface of the optimization monitoring points

It is indicated in Fig. 9 that more test data deviate from the fit left surface, but almost all of optimization points in Fig. 10 lie in the right surface.

In cases 1 and 2, the natural frequencies and mode shapes are shown in Figs. 11-14.

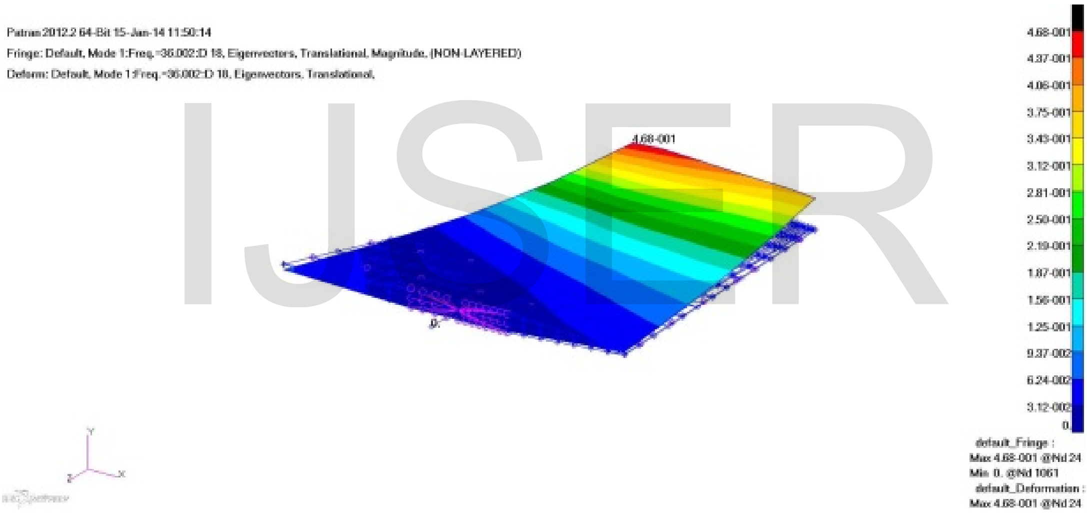


Figure 11: First bending mode (case 1)

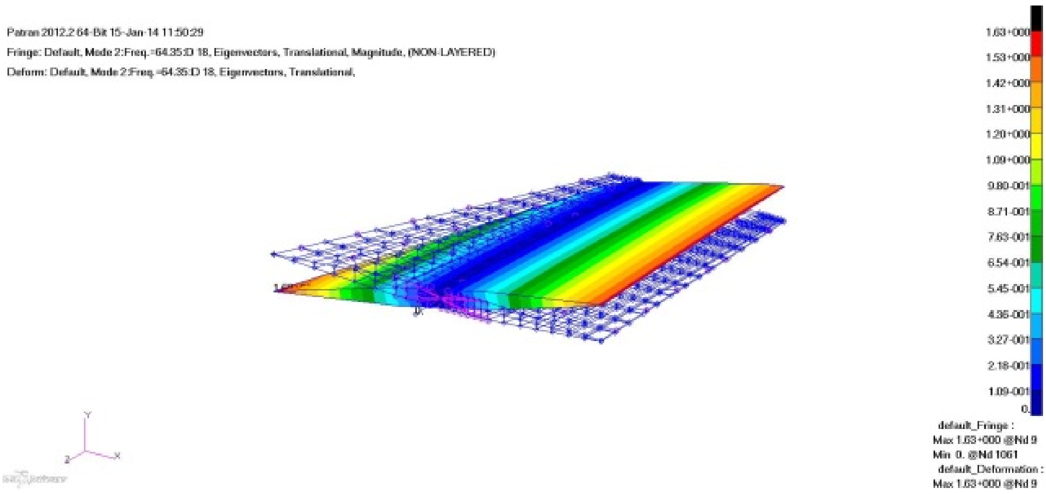


Figure 12: Second torsion mode (case 1)

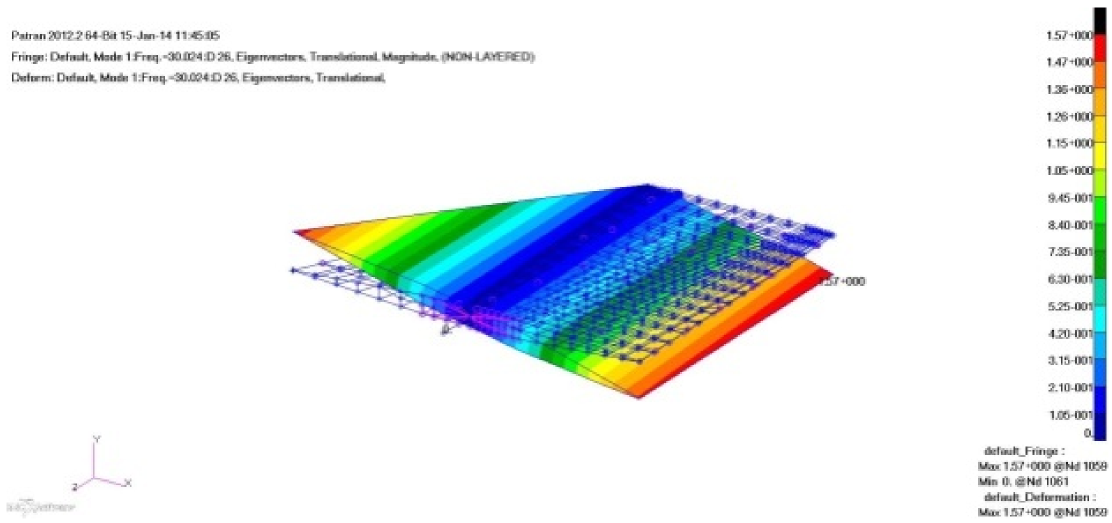


Figure 13: First torsion mode (case 2)

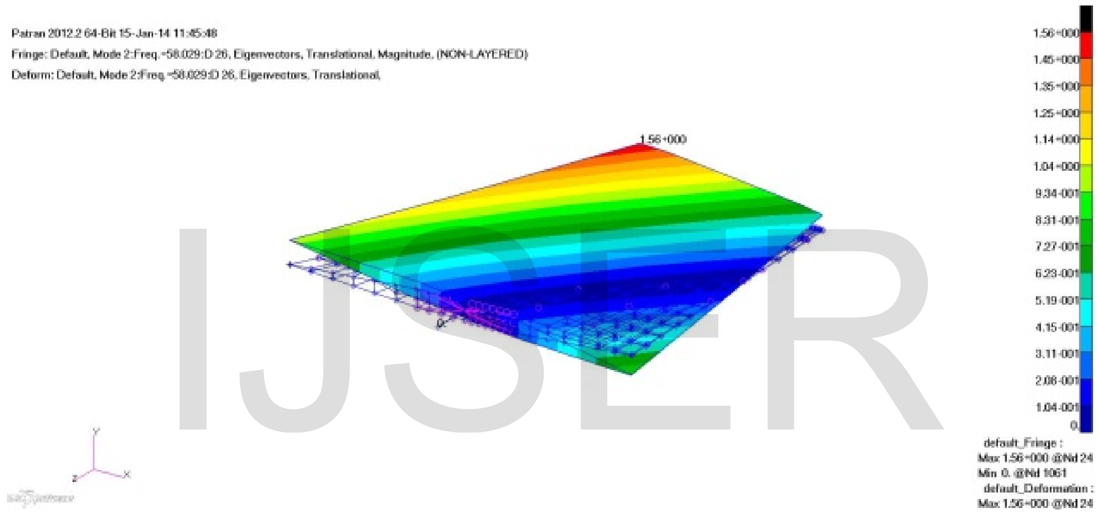


Figure 14: Second bending mode (case 2)

Compared with the experimental frequencies, the errors are shown in table 4.

Table 4: Frequency errors between test and optimization

Cases	first frequency		Error	second frequency		Error
	Test	Opti.	(%)	Test	Opti.	(%)
1	35.86	36.002	0.396	64.74	64.35	0.602
2	29.68	30.024	1.16	58.86	58.029	1.41

From Table 4, the error of the first frequency is less than 1%, and that of the second one is not more than 1.5%. Again, it represents optimization design is very successful.

4. Flutter Prediction

In this section, two flutter-prediction methods, namely Zona51of Nastran from MSC.software corporation, and Local piston theory, which is performed by home-made software are employed to obtain the flutter speeds.

ZONA51, written by MSC Software Corporation[9], is a supersonic lifting surface theory that accounts for the interference among multiple lifting surfaces. It is similar to the Doublet-Lattice method (DLM) in that both are acceleration potential methods that need not account for flow characteristics in any wake. An outline of the development of the acceleration-potential approach for ZONA51 is presented and its outgrowth from the harmonic gradient method (HGM) is described. ZONA51 is a linearized aerodynamic small disturbance theory that assumes all interfering lifting surfaces lie nearly parallel to the flow, which is uniform and either steady or gusting harmonically. As in the DLM, the linearized supersonic theory does not account for any thickness effects of the lifting surfaces.

4.1 Flutter Analysis

For aeroelastic analysis, the unsteady aerodynamic forces are obtained using Doublet Lattice for supersonic flight. The rudder section was subdivided into a lattice of 20 chordwise $\times$ 20 spanwise space vortex panels, yielding a total of 400 vortex panels. Figure 15 describes aerodynamic trapezoidal panels of the rudder.

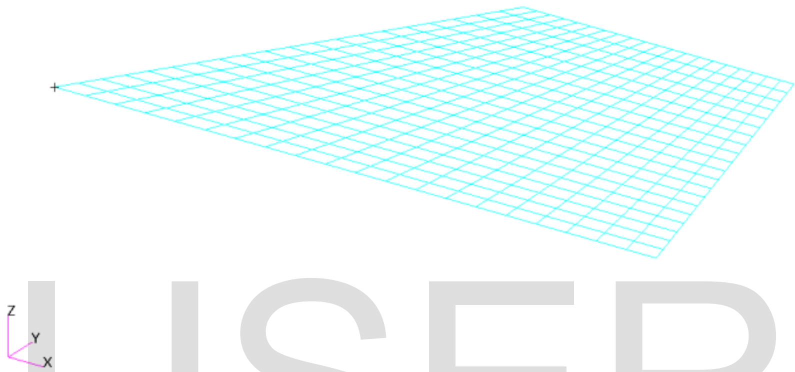
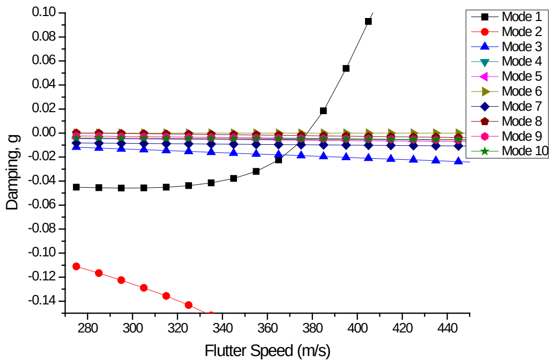


Fig. 15: DLM grid

Through the flutter analysis by Nastran's ZONA51, the V-g and V-f curves of Case 1 are shown in Figs. 16 and 17, when M=1.35 and Density Ratio=0.479.



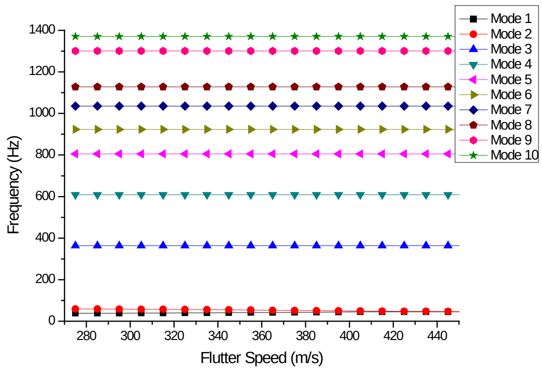
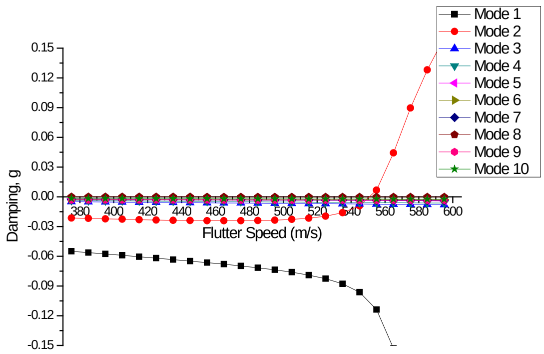


Figure 16: V-g curve Figure 17: V-f curve

From Figure 16, g of the first bending mode changes from the negative value to the positive at the speed of 380m/s, and Figure 17 presents frequencies of the second torsion mode and the first bending mode try to go toward the same value at the speed of 380m/s, that is, 1.35M. At this point, flutter occurs.

Through the flutter analysis, the V-g and V-f curves of Case 2 are shown in Figs. 18 and 19, when M=2.4 and Density Ratio=0.327.



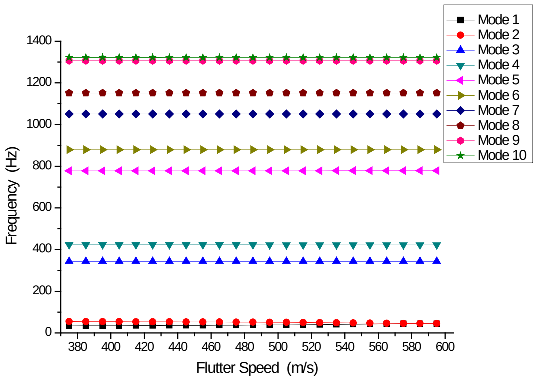


Figure 18: V-g curve Figure 19: V-f curve

From Figure 18, g of the 2st-order bending mode changes from the negative value to the positive at the speed of 550m/s, and Figure 19 presents frequencies of the second bending mode and the first torsion mode try to go toward the same value at the speed of 550m/s, e.g. 2.4M. At this point, flutter occurs.

As can be seen from the preceding figures 16-19, two cases present the same bending-torsion coupling modes that lead to flutter failure in terms of the same flutter mechanisms. However, aeroelastic flutter speeds have somewhat obvious differences, though the first two frequencies are slightly similar.

4.2 Comparison of calculated methods and tests

Due to the different aerodynamic expressions, Zona51, and Local piston theory, the flutter results are indicated in Table 5.

Table 5: Comparison of Flutter Prediction

Cases	Flutter speed (Mach)			Error (%)	
	Test	Zona51	Local piston theory	Zona51	Local piston theory
1	1.53	1.35	1.33	11.76	13.07
2	2.51	2.4	2.46	4.38	1.99

If the MAC is used this example to correct modal data, two different models cannot be obtained. From the above table, there are two different flutter speeds by use of the method which is proposed in this paper, though the results have some errors.

In figure 20, there is only two test points in Case One, which is connected to a straight line transcend the zero point, and which make us find the flutter speed. However, if the other two colorful curves have more test points, they also go through the horizontal axis, which gets the different flutter speeds, greater than the former or less than it.

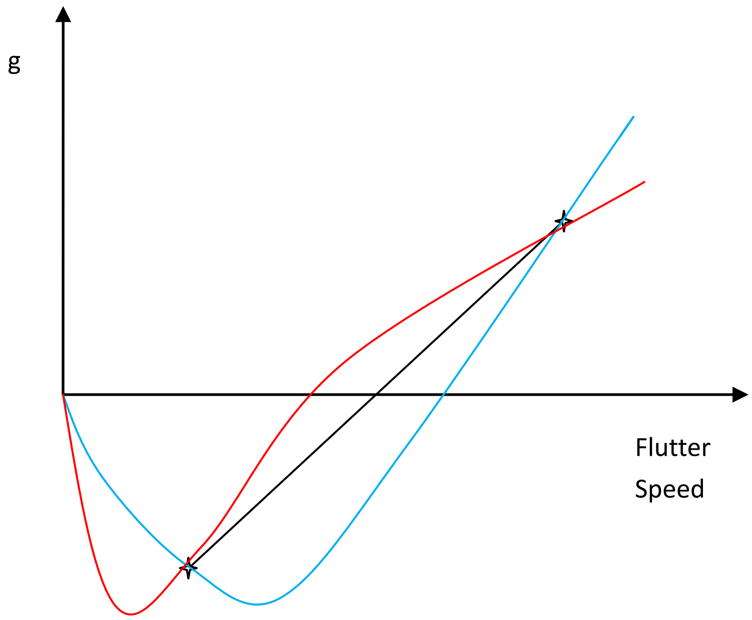


Fig. 20 Flutter V-g curve

But in Case 2, in supersonic flight, the test result is in good agreement with the methods, due to better optimization model and much more test points.

5. Conclusions

From experimental mode to simulated mode, the defined error equations of modal shapes for modal corrections have been successfully used to FEM for flutter predictions. We can draw some conclusions listed as follows:

1) When the MAC cannot judge the sequences of modes efficiently, the comparison of each modal shape is becoming very necessary. According to the experimental eigenvectors in two supersonic flight cases, optimization technique with the sensitivity-based approximation approaches, and the first-order and second-order errors, helps to find the different finite element models with the first bending and second torsion, or the first torsion and second bending, by modifying the stiffness of the rudder axle.

2). During the optimization procedure, Figures 8 and 10 represent the optimized mode surface can correct some mistake of the test data, perhaps, coming from manual errors or tool limitations.

6. Final Remarks

Using the MAC as part of the optimization, is an obvious shortcoming, which is not so sure that the modal order is consistent, namely, some modes are also likely to reverse, miss or increase, though it may guarantee the boundary still between 0 and 1. However，the outstanding advantage of this current proposal is that the difference between the vibration amplitude of the monitoring point of the modal test and the vibration amplitude of the corresponding node of the finite element is established to form the first or second order or higher order error, which is minimized as multiple optimization objectives. Thus, the order of modal calculation and experimental measurement (no modal inversion, loss or addition), frequency and vibration are consistent.

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